

# London-Paris Bachelier Workshop on Mathematical Finance 2015

The Great Hall, King's College London

## **Program: Friday 25 September 2015**

1.30 - 2.30 **Arrival, Welcome, Registration**

2.30 - 3.00

**John Armstrong** (King's College London)  
Stochastic Filtering by Projection

3.00 - 3.30

**Marie-Claire Quenez** (Université Paris Diderot)  
A Weak Dynamic Programming Principle for Mixed Optimal  
Control/Stopping Problems with  $f$ -Expectations

3.30 - 4.00 **Coffee Break**

4.00 - 4.30

**Laura Ballotta** (City University London)  
Integrated Structural Approach to Counterparty Credit Risk with  
Dependent Jumps

4.30 - 5.00

**Stéphane Crépey** (Université d'Evry)  
Central Clearing Valuation Adjustment

5.00 - 5.15 **Short Break**

5.15 - 5.45

**Elena Boguslavskaya** (Brunel University London)  
A-transforms and Lévy processes

5.45 - 6.15

**Dylan Possamai** (Université Paris Dauphine)  
Contracting Theory with Interacting Agents

6.15 - 6.30 **Short Break**

6.30 - 8.30

**Poster Presentations and Drinks Reception** (in the River Room\*)

**Nicolas Baradel** (ENSAE Paris Tech)

Optimal Control of Trading Algorithms and Bayesian Parameter Adjustments

**Yupeng Jiang** (University College London)

Real-Time Risk Management: an AAD-PDE Approach

**Gang Liu** (Ecole Polytechnique)  
Rare Event Simulation Related to Financial Risks

**Thibaut Mastrolia** (Université Paris Dauphine)  
On the Regularity of Solutions to Backward SDEs

**Stefano Pagliarani** (Ecole Polytechnique)  
Analytical Approximations of BSDEs with Non-Smooth Driver

**Jose Pasos** (London School of Economics)  
Irreversible Capacity Expansion with Possible Default

**Andrea Romeo** (City University London)  
Pricing Multivariate Barrier Reverse Convertible with Factor-Based Subordinators

**Zhenjie Ren** (Ecole Polytechnique)  
A Dual Algorithm for Stochastic Control Problems and its Applications in Finance

**Junwei Xu** (London School of Economics)  
Optimal Liquidation Trajectories for the Almgren-Chriss Model with Lévy Processes

**Yiyi Zou** (Université Paris Dauphine)  
Almost-Sure Hedging with Permanent Price Impact

\*The Poster Presentations and Reception will take place in the River Room. To get to the River Room from the Great Hall, take the stairs up from the foyer. At the top of the stairs, turn right and walk to the end of the corridor. The River Room will be on your left.

### **Program: Saturday 26 September 2015**

9.00 - 9.30 **Arrival, Welcome, Registration**

9.30 - 10.00

**Christoph Czichowsky** (London School of Economics)  
Portfolio Optimisation under Proportional Transaction Costs

10.00 - 10.30

**Aurélien Alfonsi** (Ecole des Ponts Paristech)  
Optimal Execution in a Hawkes Price Model

10.30 - 11.00 **Coffee Break**

11.00 - 11.30

**Mikko Pakkanen** (Imperial College London)  
Hybrid Scheme for Brownian Semi-Stationary Processes

11.30 - 12.00

**Claudio Fontana** (Université Paris Diderot)  
A New Martingale Representation Result in Initially Enlarged Filtrations

12.00 - 12.30

**Johannes Ruf** (University College London)  
Financial Models with Defaultable Numeraires

12.30 - 2.30 **Lunch Break** (see page 10 for some suggestions)

2.30 - 3.00

**Anthony Réveillac** (INSA Toulouse)  
Stochastic Regularization Effects of Semimartingales

3.00 - 3.30

**Justin Sirignano** (Imperial College London)  
Loan Portfolio Risk and Optimization

3.30 - 4.00 **Coffee Break**

4.00 - 4.30

**Hao Xing** (London School of Economics)  
Incomplete Stochastic Equilibria and a System of Quadratic BSDEs

4.30 - 5.00

**Emmanuel Gobet** (Ecole Polytechnique)  
Stratified Regression Monte-Carlo Scheme for BSDEs with Large-Scale Parallelization on GPUs

5.00 - 5.30 **Closing Remarks and Departure**

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Organizing Committee:

Luciano Campi (London School of Economics), Emmanuel Gobet (Ecole Polytechnique), Lane Hughston (Brunel University London), Antoine Jacquier (Imperial College London), and Teemu Pennanen (King's College London), in collaboration with the Paris Bachelier Group.

## Abstracts of talks

**Aurélien Alfonsi, Ecole des Ponts Paristech. Optimal Execution in a Hawkes Price Model.** Abstract: We study a linear price impact model including other liquidity takers, whose flow of orders follows a Hawkes process. The optimal execution problem is solved explicitly in this context, and the closed-formula optimal strategy describes in particular how one should react to the orders of other traders. This result enables us to discuss the viability of the market. Last, we will present a way to calibrate this model to market data and back test the optimal execution strategy.

**John Armstrong, King's College London. Stochastic Filtering by Projection.** Abstract: Stochastic filtering is the problem of calculating the probable state of a system from noisy measurements. Stochastic filtering frequently arises in financial applications, for example one might want to estimate the financial health of a company from the stock price and other publicly available variables or one might want to estimate an investor's financial position by observing their trading behaviour. Solving stochastic filtering problems requires solving a stochastic partial differential equation which can be very time consuming for even rather low dimensional problems. We will discuss how 'projection' can be used to approximate the problem using a finite collection of ordinary stochastic differential equations and hence obtain systems that can be calculated in real time. Our focus will be on the meaning of 'projection' for stochastic differential equations as we believe the notion of projection will have applications beyond the specific case of stochastic filtering. Joint work with D. Brigo.

**Laura Ballotta, City University London. Integrated Structural Approach to Counterparty Credit Risk with Dependent Jumps.** Abstract: This paper proposes an integrated pricing framework for counterparty credit risk: the model is based on a structural approach which uses correlated exponential Lévy processes with idiosyncratic and systematic components; the efficient numerical scheme, instead, combines Monte Carlo simulation and Fourier inversion. The framework is sufficiently flexible in incorporating a number of mitigating clauses, such as netting and collateral provisions. We illustrate the tractability and the performance of the proposed numerical scheme, and analyse the effects originated by right-way and wrong-way risk under different assumptions related to the parameters controlling collateral and netting clauses. Joint work with G. Fusai (City University London and Università del Piemonte Orientale) and D. Marazzina (Politecnico Milano).

**Elena Boguslavskaya, Brunel University London. A-transforms and Lévy processes.** Abstract: Here we introduce the so-called A-transform built on a random variable  $\eta$ . The A-transform, if applied to a monomial  $x^n$  results in a well-known Appell polynomial  $Q_n^\eta(x)$ . Not surprisingly, the transformed function has properties similar to an Appell polynomial. For example, the transformed function is a martingale if the transform is built on a Lévy process. As a consequence of the above, the A-transform is especially useful for solving problems related to Lévy processes. For instance, it gives a straightforward formula for the calculation of European-type functionals of Lévy processes. In the context of optimal stopping, one can obtain an optimal stopping rule by studying the geometrical properties of the transformed payoff. If compared to the standard approach, the A-transform method benefits from the absence of integro-differential equations, making the process of obtaining the solution much easier. We illustrate the method with some examples.

**Stéphane Crépey, Université d'Evry. Central Clearing Valuation Adjustment.**

Abstract: We develop an XVA analysis of centrally cleared trading, parallel to the one that has been developed in the last years for bilateral transactions. A dynamic framework incorporates the exact sequence of all the cash flows involved in the waterfall. The total cost of the clearance framework for a member of the clearinghouse, called CCVA for central clearing valuation adjustment, is decomposed into a nonstandard CVA corresponding to the cost of the losses on the default fund due to realized breaches, an FVA corresponding to the cost of funding the variation, initial and default fund margins and a KVA corresponding to the cost of the regulatory capital required from a member of the clearinghouse (and for completeness we also incorporate a DVA term). This framework can be used by a clearinghouse to assess the right balance between margins and default fund in order to minimize the CCVA, and hence become more competitive with respect to other clearinghouses. A clearinghouse can also use it to analyze the benefit for a dealer to trade centrally as a member, rather than on a bilateral basis, or to help its members risk manage their CCVA. Joint work with Y. Armenti, Université d'Evry and LCH.Clearnet

**Christoph Czichowsky, London School of Economics. Portfolio Optimization under Proportional Transaction Costs.**

Abstract: While absence of arbitrage in frictionless financial markets requires price processes to be semimartingales, non-semimartingales can be used to model prices in an arbitrage-free way, if proportional transaction costs are taken into account. In this talk, we show, for a class of price processes that are not necessarily semimartingales, the existence of an optimal trading strategy for utility maximization under transaction costs by establishing the existence of a so-called shadow price. This is a semimartingale price process, taking values in the bid ask spread, such that frictionless trading for that price process leads to the same optimal strategy and utility as the original problem under transaction costs. Our results combine arguments from convex duality with the stickiness condition introduced by P. Guasoni. They apply in particular to exponential utility and geometric fractional Brownian motion. In this case, the shadow price is an Ito process. As a consequence we obtain a rather surprising result on the pathwise behaviour of fractional Brownian motion: the trajectories may touch an Ito process in a one-sided manner without reflection. Joint work with W. Schachermayer.

**Claudio Fontana, Université Paris Diderot. A New Martingale Representation Result in Initially Enlarged Filtrations.**

Abstract: We provide a general account of the martingale representation property in filtrations initially enlarged with a random variable  $L$ . We prove that the martingale representation property can always be transferred to the enlarged filtration as long as the classical density hypothesis of Jacod (1985) holds. This generalizes the existing martingale representation results and does not rely on the equivalence between the conditional and the unconditional law of  $L$ . The results are illustrated in the context of hedging contingent claims under insider information.

**Emmanuel Gobet, Ecole Polytechnique. Stratified Regression Monte-Carlo Scheme for BSDEs with Large-Scale Parallelization on GPUs.** Abstract: In this paper, we design a novel algorithm based on least-squares Monte Carlo (LSMC) in order to approximate the solution of discrete-time backward stochastic differential equations (BSDEs). Our algorithm allows massive parallelization of the computations on multicore devices such as graphics processing units (GPUs). Our approach consists of a novel method of stratification which appears to be crucial for large scale parallelization. Joint work with J. G. Lopez-Salas, P. Turkedjiev and C. Vazquez.

**Mikko Pakkanen, Imperial College London. Hybrid Scheme for Brownian Semi-Stationary Processes.** Abstract: We introduce a simulation scheme for Brownian semistationary processes (Barndorff-Nielsen & Schmiegel, 2007), which form a class of stochastic processes constructed by convolving a deterministic kernel function with a Brownian motion modulated by stochastic volatility. Our scheme is based on discretizing this stochastic integral representation of the process in the time domain. We assume that the kernel function is regularly varying at zero. The novel feature of the scheme is to approximate the kernel function by a power function near zero and by a step function elsewhere. The resulting approximation of the process is a combination of Wiener integrals of the power function and a Riemann sum, which is why we call this method a hybrid scheme. Our main theoretical result describes the asymptotics of the mean square error of the hybrid scheme and we observe that the scheme leads to a substantial improvement of accuracy compared to the ordinary forward Riemann-sum scheme, while having the same computational complexity. We exemplify the use of the hybrid scheme by two numerical experiments, where we examine the finite-sample properties of an estimator of the roughness parameter of a Brownian semistationary process and study Monte Carlo option pricing in the rough Bergomi model of Bayer et al. (2015), respectively. Joint work with M. Benedsen and A. Lunde.

**Dylan Possamai, Université Paris Dauphine. Contracting Theory with Interacting Agents.** Abstract: We consider a model where a principal requires to contract separately with a large number of agents. In this framework, each agent will be in charge of one project, whose stochastic dynamics can be influenced by all the agents. More specifically, each agent can choose to make efforts towards managing his own project, but can also decide to impact (positively or negatively) the projects of the other agents. Considering agents with relative performance concerns, we look towards the optimal way for the principal to contract with the interacting agents and discuss in particular the role of competition in this framework. Joint work with R. Elie.

**Marie-Claire Quenez, Université Paris Diderot. A Weak Dynamic Programming Principle for Mixed Optimal Control/Stopping Problems with  $f$ -Expectations.** Abstract: We study mixed optimal control/stopping problems with  $f$ -expectations/evaluations in the Markovian framework on a finite horizon of time  $T$ . We establish a weak dynamic programming principle (DPP), which extends to the (nonlinear) case of  $f$ -expectations the one obtained by Bouchard & Touzi (2011) in the case of classical expectations. Using this weak DPP and properties of reflected backward stochastic differential equations, we prove that the value function of our mixed control problem, which is not necessarily continuous, not even measurable, is a weak viscosity solution of a nonlinear Hamilton-Jacobi-Bellman variational inequality.

**Anthony Réveillac, Institut National des Sciences Appliquées de Toulouse. Stochastic Regularization Effects of Semimartingales.** Abstract: In this talk we address an open question formulated by Flandoli, Gubinelli and Priola concerning an extension to random mappings  $f$  of the so-called Ito-Tanaka trick which links the time-average of a deterministic function  $f$  depending on a stochastic process  $X$  and  $F$  the solution of the Fokker-Planck equation associated to  $X$ . To this end we provide new results on a class of adapted and non-adapted Fokker-Planck SPDEs and BSPDEs.

**Johannes Ruf, University College London. Financial Models with Defaultable Numeraires.** Abstract: Financial models are studied where each asset may potentially lose value relative to any other. To this end, the paradigm of a pre-determined numeraire is abandoned in favour of a symmetrical point of view where all assets have equal priority. This approach yields novel versions of the fundamental theorems of asset pricing, which clarify and extend non-classical pricing formulas used in the financial community. Furthermore, conditioning on non-devaluation, each asset can serve as numeraire and a classical no-arbitrage condition can be formulated. It is shown when and how these local conditions can be aggregated to a global no-arbitrage condition. Joint work with T. Fisher and S. Pulido.

**Justin Sirignano, Imperial College London. Loan Portfolio Risk and Optimization.** Abstract: Financial institutions, GSEs such as Freddie Mac, and investors are exposed to default and prepayment risk from large numbers of loans. Examples include mortgages, business loans, credit cards, and other consumer loans. The estimation and management of these exposures is computationally challenging due to the large sizes of loan portfolios and high-dimensional loan-level data common in practice. For a broad class of dynamic loan-level models, we develop approximations for risk analysis and optimal selection of loan portfolios. The approximations aggregate the full loan-level dynamics as well as the specific features of each loan, making it possible to take advantage of the detailed loan-level data often available. Using a data set of 25 million actual mortgages, we compare the performance of our methods against status quo simulation and optimization approaches. Computational cost is often several orders of magnitude less with a similar level of accuracy. Joint work with K. Giesecke and G. Tsoukalas.

**Hao Xing, London School of Economics. Incomplete Stochastic Equilibria and a System of Quadratic BSDEs.** Abstract: We tackle a number of problems related to the existence of continuous-time stochastic Radner equilibria with incomplete markets. We translate the problem to a fully-coupled nonlinear system of quadratic BSDEs. In the non-Markovian setting, we establish the existence and uniqueness under various assumptions of "smallness" type, including a new notion of "closeness" to Pareto optimality. In the Markovian setting, we establish the global existence by revisiting a paper of Bensoussan & Frehse (2002). Joint work with K. Kardaras and G. Zitkovic.

## Abstracts of posters

**Nicolas Baradel, ENSAE Paris Tech. Optimal Control of Trading Algorithms and Bayesian Parameter Adjustments.** Abstract: We propose a general framework for the optimal control/design of trading algorithms in situations where the impact parameters are uncertain. Given a prior on the distribution of the unknown parameters, we explain how it should evolve according to the classical Bayesian rule after each sequence of trades. Taking these progressive prior-adjustments into account, we establish a dynamic programming principle for the value function associated to the optimal control of a trading algorithm. This leads to a characterization of the optimal policy through a quasi-variational parabolic equation, which can be solved numerically. Various examples of application are discussed.

**Yupeng Jiang (University College London) Real-Time Risk Management: an AAD-PDE Approach.** Abstract: We apply adjoint algorithmic differentiation (AAD) to the risk management of securities when their price dynamics are given by partial differential equations (PDEs). We show how AAD can be applied to forward and backward PDEs in a straightforward manner. In the context of one-factor models for interest rates or default intensities, we show how price sensitivities are computed reliably and orders of magnitude faster than with a standard finite-difference approach. This significantly increased efficiency is obtained by combining (i) the adjoint forward PDE for calibrating model parameters, (ii) the adjoint backward PDE for derivatives pricing, and (iii) the implicit function theorem to avoid iterating the calibration procedure.

**Gang Liu, Ecole Polytechnique. Rare Event Simulation Related to Financial Risks.** Abstract: We present two different methods to evaluate extreme financial risks such as systemic default and large hedging loss. Both methods are based on a random transformation called reversible shaking transformations. We also present some variants of these methods and discuss about how to make appropriate choices of parameters to achieve good numerical performances.

**Thibaut Mastrolia, Université Paris Dauphine. On the Regularity of Solutions to Backward SDEs.** Abstract: In this work, we study the Malliavin differentiability of solutions  $(Y,Z)$  to Lipschitz or quadratic growth BSDEs. Incidentally, this question lead us to a new characterization of Malliavin-Sobolev spaces. Joint work with D. Possamaï and A. Réveillac, and with P. Imkeller, D. Possamaï and A. Réveillac.

**Stefano Pagliarani, Ecole Polytechnique. Analytical Approximations of BSDEs with Non-Smooth Driver.** Abstract: We provide and analyze analytical approximations of BSDEs in the limit of small non-linearity and short time, in the case of non-smooth drivers. We identify the first and the second order approximations within this asymptotics and consider two topical financial applications: the two interest rates problem and the Funding Value Adjustment. In high dimensional diffusion setting, we present numerical tests to illustrate the efficiency of the numerical schemes. Finally, we discuss the limit of this approach by assessing the possibility of higher order expansions.

**Jose Pasos, London School of Economics. Irreversible Capacity Expansion with Possible Default.** Abstract: We consider an irreversible capacity expansion model in which an economic indicator is driven by a linear diffusion. The investor can increase the



project's capital until default or bankruptcy time is reached, which is modeled by the hitting time of zero. The associated optimization problem takes the form of a singular stochastic control problem which, under suitable assumptions, admits explicit solution.

**Andrea Romeo, City University London. Pricing Multivariate Barrier Reverse Convertible with Factor-Based Subordinators.** Abstract: The paper studies the ability of factor-based subordinated Lévy processes in pricing multivariate exotic derivatives. Different model specifications, calibrated to a dataset of multivariate Barrier Reverse Convertible at the Swiss market, show diverse ability in capturing smile patterns and recovering empirical correlations. Our analysis finds that there exists a trade-off between marginal and dependence fit. Return distributions of different model specifications are compared. A sensitivity analysis is performed, showing how product and model's features affect multi-barrier reverse convertible prices. Market and model prices are analyzed, highlighting and explaining discrepancies.

**Zhenjie Ren, Ecole Polytechnique. A Dual Algorithm for Stochastic Control Problems and its Applications in Finance.** Abstract: When one solves a stochastic control problem by a regression algorithm based on Monte-Carlo simulation, she will get a lower biased numerical solution. To make the numerical solution more reliable, we would like to find an upper bound for the value function. In recent work with P. Henry-Labordère and C. Litterer, we derive a dual algorithm providing such an upper bound, and show some applications in finance.

**Junwei Xu, London School of Economics. Optimal Liquidation Trajectories for the Almgren-Chriss Model with Lévy Processes.** Abstract: We consider an optimal liquidation problem with infinite horizon in the Almgren-Chriss framework, where the unaffected asset price follows a Levy process. The temporary price impact is described by a general function that satisfies some reasonable conditions. We consider an investor with constant absolute risk aversion, who wants to maximize the expected utility of the cash received from the sale of his assets, and show that this problem can be reduced to a deterministic optimization problem that we are able to solve explicitly. In order to compare our results with exponential Levy models, which provide very good statistical fit with observed asset price data for short time horizons, we derive the (linear) Levy process approximation of such models. In particular we derive expressions for the Levy process approximation of the exponential variance-gamma Lévy process, and study properties of the corresponding optimal liquidation strategy. We find that for the power-law temporary impact function, the optimal strategy is to liquidate so quickly that it may be practically impossible. We therefore study the case where the temporary impact function follows a power-law for small liquidation speeds, but tends faster to infinity than a power-law as the liquidation speed tends to infinity. In particular, we obtain an explicit expression for the connection between the temporary impact function for the Levy model and the temporary impact function for the Brownian motion model, for which the optimal liquidation strategies from the two models coincide. Joint work with A. Lokka.

**Yiyi Zou, Université Paris Dauphine. Almost-Sure Hedging with Permanent Price Impact.** Abstract: We consider a financial model with permanent price impact. Continuous time trading dynamics are derived as the limit of discrete rebalancing policies. We then consider the problem of super-hedging a European option. Our main result is the derivation of a quasi-linear pricing equation. It holds in the sense of viscosity solutions. When it admits a smooth solution, it provides a perfect hedging strategy.

### **Lunch break on Saturday 12.30 – 2.30**

There are plenty of nearby restaurants and cafes on the Strand and in the Covent Garden area. Here is a list of some English pubs in the area that usually serve food at lunchtime:

Great Queen Street Restaurant  
32 Great Queen Street  
London WC2B 5AA

The Knights Templar  
95 Chancery Ln  
London WC2A 1DT

The Ship Tavern  
12 Gate Street  
London WC2A 3HP

Seven Stars  
53-54 Carey Street  
London WC2A 2JB

The Edgar Wallace  
40 Essex Street  
London WC2R 3JE

The Temple Brew House  
46 Essex Street  
London WC2R 3JF

The Old Bank of England  
194 Fleet Street  
London EC4A 2LT

Ye Olde Cheshire Cheese  
145 Fleet Street  
London EC4A 2BU

The Princess Louise  
208 High Holborn  
London WC1V 7EP

There are of course many other options.